ON THE KINETICS OF CRACK GROWTH

(K KINETIKE ROSTA TRESHCHIN)

PMM Vol.25, No.3, 1961, pp. 498-502

L.M. KACHANOV
(Leningrad)

(Received March 13, 1961)

According to present opinion, fracture is a process that develops in time. Fracture depends on the character of the loads, which may be of a constant, cyclic, impulsive or mixed type. Here we shall limit ourselves to the analysis of fracture under the action of constant loads.

From the very beginning, there are a great number of different defects distributed throughout the body, which are embryonic cracks. During loading, cracks arise in the weakened spots of the body. The simplest case of a straight isolated crack in a plane homogeneous field of an elastic body was studied by many authors – Griffith [1], Orowan [2], Irwin [3], and others. General results were recently obtained in the papers by Barenblatt [4-8]. During the widening of a crack, some elastic energy is released which during a slow crack opening is spent on the work of the surface forces and the formation of the edges of the crack. In [6] it was shown that the surface energy (for brittle materials) or the plastic surface work (for metals) is related to the cohesion modulus K. The above-mentioned papers are concerned with the analysis of equilibrium cracks, i.e. cracks whose dimensions remain unchanged under a given load. A growth of such cracks is only possible with an increase of the load.

Numerous observations, however, show that with a fixed load cracks do grow. This circumstance is one of the main reasons for the dependence of the ultimate strength on time. The latter problem (that of the so-called sustained strength) is of great practical importance, and is presently attracting the attention of many scientists and engineers. In the present paper an attempt is being made to develop an analysis of crack growth within the framework of continuum mechanics.

1. Fundamental considerations. According to observations, at first there occurs a slow growth of a crack. This stage makes up the principal part of the total life time of the element. Subsequently, there occurs an accelerated growth of the crack, which in the final stage goes into a propagation of the crack with a velocity comparable to sound velocity. It should be noted also that the problems of the development of a crack in a real body are connected with local properties of its structure and in many respects remain unclear. Nevertheless, it seems that some general rules of crack development in the first and second stages can be set forth, disregarding structural details.

In an ideally elastic body, the development of a crack is impossible with a fixed load. Thus, the growth of a crack must be related to elastic imperfections, in particular to the appearance of flow in solid bodies.

Let us study at first the case of linear creep, corresponding to the model of a Maxwell body. In this medium, the strain rates $\xi_x, \xi_y, \ldots, \eta_{xz}$ are related to the stress components $\sigma_x, \sigma_y, \ldots, \tau_{xz}$ by the relations

$$2\xi_x = \left(\frac{1}{\mu} + \frac{1}{G}\frac{d}{dt}\right)(\sigma_x - \sigma), \qquad \eta_{xy} = \left(\frac{1}{\mu} + \frac{1}{G}\frac{d}{dt}\right)\tau_{xy} \tag{1.1}$$

where μ is the viscosity coefficient, G is the shear modulus, and σ is the mean pressure. For simplicity, it was assumed that Poisson's ratio equals one-half.

Following Barenblatt [4], we shall distinguish between two regions in the crack: the internal region, where the opposite edges of the crack have separated and there is no attraction between them, and the end région, where there exist cohesion forces. The end region d (Fig. 1) is small in comparison to the internal region (hypothesis I). The configuration of the edges of the crack in the end region does not depend on the acting loads and for a given material is always the same under given conditions (hypothesis II).

We shall also assume that the condition of finiteness of stresses at the ends of the crack and the smoothness of the closure of its edges is maintained (hypothesis III).

Let us study, for definiteness, the states of stress and strain of an infinite plate with a straight crack running from $x = a$ to $x = b$. The stress distribution in an elastic plate with given loads does not depend on the elastic modulus, with arbitrary values of abscissas of the crack ends a, b. Then the same stress field, according to Volterra's principle [9], will exist also in a linear visco-elastic plate.

This remains the case also with a change of some dimensions of the body (for instance, the crack length) with time. The displacement in a visco-elastic plate is equal to

$$\mathbf{u} = \mathbf{u}_0 + G/\mu \int_0^t \mathbf{u}_0\, dt \tag{1.2}$$

where $\mathbf{u}_0$ is the displacement in the elastic state. With fixed dimensions $\mathbf{u}_0$ does not depend on time, and then

$$\mathrm{u} = \mathrm{u}_0 \left(1 + \frac{G}{\mu} t\right) \tag{1.3}$$

Since the crack configuration in the end region is preserved, relation (1.3) characterizes the rate of opening of this region, if t is chosen to be a short time after the arrival of the crack at a given point.

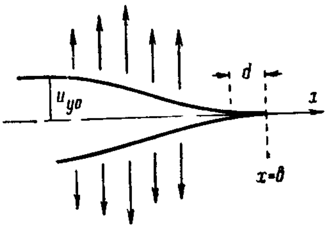

Fig. 1

Thus, in a visco-elastic body the edges of a crack spread apart with time, whereas the ends close smoothly.

As shown by Barenblatt [4,5], the size of the crack in an elastic body is given by the relation

$$\Phi(l) = \frac{K}{\sqrt{2\lambda}} \tag{1.4}$$

where λ is the load parameter. Function $\Phi(l)$ depends on the type of crack and is related essentially to the formation of a given crack. The cohesion modulus K is equal to

$$K = \int_0^d \frac{F(s)\,ds}{\sqrt{s}} \tag{1.5}$$

Here the function $F(s)$ characterizes the distribution of the tensile stresses, which depend on the cohesion forces. In the elastic body the distance between the edges of the crack does not change for a given load.

Relations (1.4) and (1.5), which express the finiteness condition of the stresses at the ends of the crack, hold also in the case of the presently studied linear visco-elastic body. As was stated above, however, the edges of the crack move apart with time. Thus, with a fixed loading and a fixed crack size the distance between its edges also increases at the end zone. This should be related, generally speaking, to a decrease of K due to a decrease of the cohesion forces. Together with this a widening of the crack takes place, which leads to a re-establishment of the previous value of K, etc.

Equation (1.5) essentially represents the equilibrium condition. The rate of decrease of K with a fixed size of crack is characterized by the value of the local derivative at initial time

$$(\partial K / \partial t)_{t=0}$$

This change is compensated by a corresponding change of the left-hand side, i.e.

$$\Phi'(l)\frac{dl}{dt} = \frac{1}{\sqrt{2}\lambda}\left(\frac{\partial K}{\partial t}\right)_{t=0} \tag{1.6}$$

Let us study some formulations concerning the derivative on the right-hand side. As it was emphasized by Barenblatt [4], the shape of the end region corresponds to that of the highest possible resistance. The separation of the edges in the end region leads to a decrease of the resistance, and for short times (for a crack of fixed size) we have

$$K = (K)_{t=0} - \varkappa\frac{G}{\mu}t + \ldots \tag{1.7}$$

where the coefficient $\kappa > 0$ can be regarded as some new material constant.

This characteristic will be very important from the practical point of view, since for considerations of the strength of the material the presence of flaws (defects, cracks) is not as essential as their eventual development. Let us call κ the flaw coefficient.

Thus, according to (1.6) and (1.7), we obtain

$$\Phi'(l)\frac{dl}{dt} = -\frac{G}{\mu}\frac{\varkappa}{\sqrt{2}\lambda}$$

From this we find

$$\Phi(l) - \Phi(l_0) = -\frac{G}{\mu}\frac{\varkappa}{\sqrt{2}\lambda}t \tag{1.8}$$

where $2l_0$ is the size of the crack in the elastic body. Relation (1.8) can be also written as

$$\Phi(l) = \frac{1}{\sqrt{2}\lambda}\left(K - \varkappa\frac{G}{\mu}t\right) \tag{1.9}$$

Here, and in the remainder of the paper, K is understood to be the value of the cohesion coefficient, as determined by Barenblatt.

If the crack is stable [6], i.e. in order to increase the size of the crack, it is necessary to increase the load, $\Phi'(l) < 0$ and then $dl/dt > 0$, i.e. the crack grows with time.

If the crack is unstable, $\Phi'(l) > 0$ and consequently $dl/dt < 0$. This result should be interpreted as follows: the flow of the material introduces a disturbance into the equilibrium state and the unstable crack widens catastrophically.

2. Example. As illustrations, let us study two simple examples.

1) A crack is formed in a plane field, caused by two equal forces P which act in opposite directions along a straight line. It can be easily seen that [5]

$$\Phi(\xi) = \frac{1}{\sqrt{L}} \frac{(1.75+\xi^2)^2 \sqrt{\xi}}{(1+\xi^2)^{3/2}} \qquad \left(\xi = \frac{l}{L}\right)$$

where $2L$ is the distance between the points of application of the forces. The load parameter is $\lambda = P$. The time dependence of ξ is shown in Fig. 2. At the initial time $\xi = \xi_0$ and at time $t_* = K\mu / \kappa G$ there occurs a catastrophic widening of the crack. Thus, there exists a finite time of fracture. If the force P is smaller than the critical value, the crack does not appear.

2) The edges of a crack in an infinite slab of width $2L$ are separated by concentrated forces P equal in magnitude and opposite in direction. In this case [5]

$$\Phi(\xi) = \sqrt{\frac{\pi}{4L}} \frac{1}{\sin \pi\xi}$$

In time, the crack develops in the following manner. During loading by a force $P < P_{max}$ an equilibrium crack is formed. Beyond that, it grows gradually, at time

$$t_1 = \frac{\mu}{\kappa G}\left(1 - \sqrt{\frac{\pi}{2L}} \frac{P}{K}\right)$$

the crack becomes unstable, and there then occurs a rapid fracture.

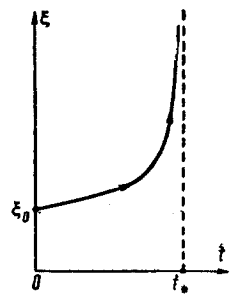

Fig. 2.

3. **Conclusion.** The qualitative concept studied above is preserved in general also for other linear media that possess a flow property. Assume, for instance, that a material follows Boltzmann's integral relations

$$\varepsilon_* = \frac{1}{2G_*}(\sigma_* - \sigma), \qquad \gamma_{xy} = \frac{1}{G_*} \tau_{xy} \tag{3.1}$$

Here the operator is introduced

$$\frac{1}{G_*} f = \frac{1}{G} f + \int_0^t M(t-s) f(s)\, ds$$

with an after-effect kernel $M(t - s)$. According to Volterra's principle, the state of stress is the same as in the elastic body.

For a short time t after the arrival of the crack at a given point in the end region, the displacement is equal to

$$\mathbf{u} = \mathbf{u}_0 [1 + GM(t)\tau] \tag{3.2}$$

Substitute

$$M^\circ(t) = G\int_0^t M(t-s)\,ds$$

After analysing an equation of the form (1.6), we easily obtain instead of (1.9) the following relation:

$$\Phi(l) = \frac{1}{\sqrt{2\lambda}}[K - \varkappa M^\circ(t)] \tag{3.3}$$

If the effect of time has the character of a negligible damped after-effect (i.e. $M^\circ(t) \to M^\circ(\infty)$ as $t \to \infty$, where $M^\circ(\infty) \ll 1$) then, after opening, the stable crack will widen for some time, but soon its development will stop for all practical purposes. On the other hand, with an undamped flow (creep) the growth of the crack does not slow down. The rate of growth of the crack is determined, in particular, by the character of the after-effect kernel.

Some other effects can be explained by non-homogeneity and the influence of the deformation. For instance, a creep deformation facilitates the appearance of cracks [10]; thus the crack which approaches a given point in some interval of time encounters a smaller resistance. This effect can be taken into account by considering K and κ to be functions of the previous creep deformation.

The results studied above refer to linear media. For nonlinear flows (for instance, the case of creep in metals) the qualitative concept is generally maintained; however, it is difficult to develop quantitative characteristics, since in course of time the state of stress of the body can deviate considerably from the elastic state of stress.

BIBLIOGRAPHY

1. Griffith, A., The phenomenon of rupture and flow in solids. *Phil. Trans. Roy. Soc.* A21, 1920.

2. Orowan, E., Energy criteria of fracture. *Welding Journ., Res. Suppl.* March 1955.

3. Irwin, G., Analysis of stresses and strains near the end of a crack traversing a plate. *J. Appl. Mech.* Vol. 24, No. 3, 1957.

4. Barenblatt, G.I., O ravrovesnykh treshchinakh, obrazuiushchikhsia pri khrupkom razrushenii. Obshchie predstavleniia i gipotezy. Ose-simmetrichnye treshchiny (On equilibrium cracks occuring in brittle fracture. General concepts and hypotheses. Axisymmetric cracks). *PMM* Vol. 23, No. 3, 1959.

5. Barenblatt, G.I., O ravnovesnykh treshchinakh, obrazuiushchikhsia pri khrupkom razrushenii. Priamolineinye treshchiny v ploskikh plastinkakh (On equilibrium cracks occuring in brittle fracture. Rectilinear cracks in thin plates). *PMM* Vol. 23, No. 4, 1959.

6. Barenblatt, G.I., O ravnovesnykh treshchinakh, obrazuiushchikhsia pri khrupkom razrushenii. Ustoichivost' izolirovanykh treshchin. Sviaz' s energeticheskimi teoriami (On equilibrium cracks occuring in brittle fracture. Stability of isolated cracks. Connections with energetic theories). *PMM* Vol. 23, No. 5, 1959.

7. Barenblatt, G.I. and Cherepanov, G.P., O rasklinivanii khrupkikh tel (On the wedging of brittle bodies). *PMM* Vol. 24, No. 4, 1960.

8. Barenblatt, G.I. and Cherepanov, G.P., O ravnovesii i rasprostranenii treshchin v anizotropnoi srede (On the equilibrium and propagation of cracks in an anisotropic medium). *PMM* Vol. 25, No. 1, 1961.

9. Volterra, V., *Lecons sur les fonctions de lignes*. Paris, Gauthier-Villars, 1913.

10. Salli, A., *Polzuchest' metallov i zharoprochnye splavy (Creep of Metals and Heat Treated Alloys)*. Oborongiz, 1953.

Translated by M.I.Y.